Layered Graphene for Enhanced Transmission at Low Terahertz

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OUTLINE

- Introduction and Motivation
- Graphene-Dielectric Stack
  - Enhanced Transmission at low-THz
- Dual Capacitive/Inductive Nature of Graphene Patches
- Results and Discussions
- Conclusions
**BACKGROUND AND MOTIVATION (1)**

Induced transparency in the optical regime

- **Extremely thin** conducting layers are **almost opaque.**


- **However**, multilayer metal-dielectric PBG-like structures **become transparent** within certain frequency bands in the optical regime.

**BACKGROUND AND MOTIVATION (2)**

**Microwave transmissivity of a metamaterial-dielectric stack**

Can a similar effect be observed at microwaves??

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**Feng et al., Phys. Rev. B., 82, 085117(2005)**

**Scalora et al., Jour. App. Physics, 83, 2377–2383 (1998).**

The metal films are substituted by perforated metal layers

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**Yakovlev et al., 3rd Int. Congress on Advan. Electromag. Materi. in Microwa. and Optic.,(2009)**

BACKGROUND AND MOTIVATION (3)

What is the nature of these resonances??
- can we tune them
- can we predict the band

The number of transmission peaks is equal to the number of layers (resonators)

Kaipa et al., Opt. Express, 18, 174101 (2010)
Graphene-Dielectric Stack

Atomically thin graphene sheet

Dielectric slab
Surface Conductivity of Graphene

Surface conductivity of graphene [Kubo formula]

\[
\sigma(\omega, \mu_c, \Gamma, T) = \frac{je^2(\omega - j\Gamma)}{\pi\hbar^2} \left[ \frac{1}{(\omega - j\Gamma)^2} \int_0^\infty \left( \frac{\partial f_d(\varepsilon)}{\partial \varepsilon} - \frac{\partial f_d(-\varepsilon)}{\partial \varepsilon} \right) \varepsilon \, d\varepsilon \right] 
- \int_0^\infty \frac{f_d(-\varepsilon) - f_d(\varepsilon)}{(\omega - j\Gamma)^2 - 4(\varepsilon/\hbar)^2} \, d\varepsilon
\]

\[\sigma_{\text{intra}} = -j \frac{e^2k_B T}{\pi\hbar^2(\omega - j\Gamma)} \left( \frac{\mu_c}{k_B T} + 2\ln\left(e^{-\mu_c/k_B T} + 1\right) \right)\]

\[\sigma_{\text{inter}} = \frac{-je^2}{4\pi\hbar} \ln\left( \frac{2|\mu_c| - (\omega - j\Gamma)\hbar}{2|\mu_c| + (\omega - j\Gamma)\hbar} \right)\]

- \(e\): charge of electron, \(T\): temperature, \(\varepsilon\): energy
- \(\omega\): angular frequency, \(\hbar = h/2\pi\): reduced Planck’s constant
- \(\mu_c\): chemical potential, \(\Gamma\): phenomenological scattering rate

In the far-infrared regime, the contribution due to the interband electron transition is negligible

\[Z_s = 1/\sigma\], which at low-terahertz frequencies behaves as a low-loss inductive surface.

G. W. Hanson, J. Appl. Phys., 103, 064302 (2008)
SURFACE CONDUCTIVITY OF GRAPHENE

\[ \mu_c = 0.2 \text{ eV} \]

\[ \mu_c = 0.5 \text{ eV} \]

\[ \Gamma = 1/\tau = 1.32 \text{ meV}, \quad \tau = 0.5 \text{ ps}, \quad T = 300 \text{ K} \]

\[ \sigma_{\text{min}} = \pi e^2 / 2h = 6.085 \times 10^{-5} \text{ S} \]

Solid lines: approximate closed-form expressions (intraband + interband)
Dashed lines: numerical integration [Kubo formula]
Single sheet of graphene is highly reflective at low-THz frequencies. Behaves similar to an Inductive grid (metallic meshes) at microwaves.

Reflectivity and Transmissivity for normal incidence

\[ \Gamma = \frac{1}{\tau} = 1.32 \text{ meV} \]
\[ \tau = 0.5 \text{ ps} \]
\[ T = 300 \text{ K} \]
\[ \mu_c = 1 \text{ eV} \]
TWO-SIDED GRAPHENE STRUCTURE

Transmission resonance appears at low frequencies
FP-type resonance of dielectric slab loaded with graphene sheets
Graphene sheets effectively increase the electrical length

Thickness \((h)\): 10 \(\mu m\)
Permittivity: 10.2

\[ \Gamma = \frac{1}{\tau} = 1.32 \text{ meV} \]
\[ \tau = 0.5 \text{ ps}, T = 300 \text{ K} \]

\[ \mu_c = 0.5 \text{ eV} \]
The number of transmission peaks is equal to the number of dielectric slabs within the characteristic frequency band.

Thickness ($h$): 10 μm
Permittivity: 10.2

$\Gamma = 1/\tau = 1.32$ meV
$\tau = 0.5$ ps, $T = 300$ K

$\mu_c = 1.0$ eV
**POWER TRANSMISSION SPECTRA**

- Enhanced transmission at low-THz
- Fabry-Perot resonances of the individual open/coupled cavities

4 layer graphene structure
- 4 dielectric slabs
- 5 graphene sheets

Thickness ($h$): 10 μm
Permittivity: 10.2

$\Gamma = \frac{1}{\tau} = 1.32$ meV
$\tau = 0.5$ ps, $T = 300$ K
Electric Field Distributions

\[ \mu_c = 1.0 \text{ eV} \]
**Electric Field Distributions - Animation Plots**

Mode B

Mode D
**BRILLOUIN DIAGRAMS – PASSBANDS AND STOPBANDS**

**Multi-layer graphene-dielectric stack**

- **SB**: StopBand
- **PB**: PassBand

**Parameters:**
- Thickness ($h$): 10 μm
- Permittivity: 10.2

**Equation:**
\[ \mu_c = 1.0 \text{ eV} \]
Graphene Thick Slabs Brillouin Diagrams

Four-layer graphene-dielectric stack

- $\mu_c = 1.0 \text{ eV}$
- SB: StopBand
- PB: PassBand

- A thick dielectric slab is sometimes needed for mechanical handling
- Exhibits a series of bandpass regions separated by bandgaps

Thickness ($h$): 150 $\mu$m
Permittivity: 2.2
BROADBAND PLANAR FILTERS

- $\mu_c = 0.5 \text{ eV}$
- $\mu_c = 1 \text{ eV}$

- Number of peaks correspond to number of layers (N)
- With increase in ‘N’, all peaks lie in a characteristic frequency band
- Acts as a Wideband Bandpass filter

Graphene Sheets

\[ h = 10\mu m \]

\[ \varepsilon_r = 1 \]
Five-layer graphene/meshgrid stacks separated by free-space

- \( h: 30 \, \mu m \),
- Period \( (D) = 20 \, \mu m \),
- Strip width \( (w) = 2 \, \mu m \),
- \( t = 0.4 \, \mu m \),
- Dielectric permittivity: 1

- Graphene-air stack mimics the behavior of Fishnet-air stack at THz
Layered Graphene Patches

- Metallic patches
- Metallic mesh-grids
- Graphene sheets

- Capacitive surface reactance
  - Low-frequency passband followed by alternating stop and passbands as frequency increases

- Inductive surface reactance
  - Low-frequency stopband, followed by alternating pass and stopbands as frequency increases

- Microwave/low-terahertz frequencies

- Low-loss inductive surface reactance
  - Low-frequency stopband, followed by alternating pass and stopbands as frequency increases

- Microwave/low-terahertz frequencies

Advantage of the Graphene Metasurface

Combining the properties of the above three structures in a single configuration

Low-loss inductive and capacitive surface reactance

Combined filtering properties of the above three configurations

Low-terahertz frequencies
SURFACE IMPEDANCE OF GRAPHENE PATCHES

\[ Z_s = Z_{s1} + Z_{s2} \]
\[ = \frac{D}{(D - g)\sigma} - j \frac{\pi}{2\omega\varepsilon_0\varepsilon_r^{qs}} D \ln \{ \csc \left( \frac{\pi g}{2D} \right) \} \]

series R-L-C circuit

\[ Z_{s1} \quad \{ \text{series R-L} \} \quad Z_{s2} \quad \{ -j/(\omega C_{\text{eff}}) \} \]

\[ C_{\text{eff}} = (2/\pi)\varepsilon_0\varepsilon_r^{qs} D \ln \{ \csc [\pi g/(2D)] \} \]

\[ \varepsilon_r^{qs} = \varepsilon_r \quad \text{For interior patches} \]
\[ \varepsilon_r^{qs} = (\varepsilon_r + 1)/2 \quad \text{For patches at top and bottom interfaces} \]
For the graphene patches, the surface impedance changes from capacitive to inductive as frequency increases

- At **low frequencies** the behavior of graphene patches is similar to that of the metallic patches (capacitive)
- At **high frequencies** the behavior of graphene patches becomes similar to that of a graphene sheet (inductive)
Five-Layer Stack of Graphene Patches

\[ D = 10 \mu m; \ g = 1 \mu m; \ \varepsilon_r = 4; \ h = 10 \mu m \]

The analytical results are in good agreement with the full-wave simulations.
**TRANSMISSIVITY**

**Fig. 1**

\[ D = 10 \, \mu m; \, g = 1 \, \mu m; \, \varepsilon_r = 4; \, h = 10 \, \mu m \]

**Fig. 2**

**Fig. 3**

**Fig. 4**

**\( \mu_c = 0.5 \, eV \)**

**\( \mu_c = 1 \, eV \)**
Analytical Results

\[ D = 10 \mu m; \quad g = 1 \mu m; \quad \varepsilon_r = 4; \quad h = 10 \mu m \]

- One can clearly notice the low passband (starting from zero frequency), followed by a deep stopband, and then a second passband.

- Also, it can be noticed that with an increase in the number of layers, the number of transmission peaks which corresponds to the number of coupled layers increases, still maintaining the same characteristic frequency bands.
**Bloch-Wave Analysis**

The Brillouin diagrams perfectly predict the passband and stopband regions of the corresponding finite-layer structures.

\[ \cos(k_b h) = \cos(\theta) + j \frac{Z_d}{2Z_s} \sin(\theta) \]
For mode A, the field value is zero near the middle graphene patch, which is consistent with the electric-field distribution shown in the inset.

- For mode B, the field value is low in the middle graphene patch and is concentrated more near the remaining graphene patches, which is consistent with the electric-field distributions shown in the inset.

\[ D = 10 \, \mu m; \, g = 1 \, \mu m; \, \varepsilon_r = 4; \, h = 10 \, \mu m; \, \mu_c = 0.5 \, eV \]
**SURFACE IMPEDANCE AND TRANSMISSIVITY THIN-METAL PATCHES**

\[ D = 100 \text{ nm}; \ g = 10 \text{ nm}; \ \varepsilon_r = 1; \ h = 166 \text{ nm} \quad \text{Four-layer thin-metal patch-dielectric stack} \]

The results are calculated using a fit based on measured data [1] for the permittivity, utilizing an augmented Drude model [2]

CONCLUSIONS

- We mimic the enhanced transmission at optical frequencies with a metal-dielectric stack and in the microwave regime with stacked-metascreens, at low-THz using stacked-graphene.

- The range of frequencies where the peaks are expected for a finite graphene-dielectric stacked structure can be analytically and accurately estimated from the Bloch analysis.

- Graphene patches have a dual (capacitive and inductive) nature at low-terahertz frequencies.

- At low-frequencies, it behaves similar to that of the multilayer stack of metallic patches (capacitive, wherein the geometric patch capacitance dominates over the weak metal inductance), and at higher frequencies similar to that of a multilayer stack of contiguous graphene sheets (inductive due to a large kinetic inductance of the material), thus combining both the properties in a single configuration.